Small counts in nested Karlin's occupancy scheme generated by discrete Weibull-like distributions

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Introduction. A nested Karlin's occupancy scheme is a symbiosis of classical Karlin's balls-in-boxes scheme and a weighted branching process. To define it, imagine a deterministic weighted branching process in which weights of the first generation individuals are given by the elements of a discrete probability distribution. For each positive integer j, identify the jth generation individuals with the jth generation boxes. The collection of balls is one and the same for all generations, and each ball starts at the root of the weighted branching process tree and moves along the tree according to the following rule: transition from a mother box to a daughter box occurs with probability given by the ratio of the daughter and mother weights.

Main results. Assume that there are n balls and that the discrete probability distribution responsible for the first generation is Weibull-like. Denote by $\mathcal{K}_n^{(j)}(l)$ and $\mathcal{K}_n^{*(j)}(l)$ the number of the jth generation boxes which contain at least l balls and exactly l balls, respectively. We prove functional limit theorems (FLTs) for the matrix-valued processes $\big(\mathcal{K}^{(j)}_{[\mathrm{e}^{T+\cdot}]}(l)\big)_{j,l\in\mathbb{N}}$ and $\big(\mathcal{K}^{*(j)}_{[\mathrm{e}^{T+\cdot}]}(l)\big)_{j,l\in\mathbb{N}}$, properly normalized and centered, as $T\to\infty$. The present FLTs are an extension of a FLT proved by Iksanov, Kabluchko and Kotelnikova (2022) for the vector-valued process $\big(\mathcal{K}^{(j)}_{[\mathrm{e}^{T+\cdot}]}(1)\big)_{j\in\mathbb{N}}$. While the rows of each of the limit matrix-valued processes are independent and identically distributed, the entries within each row are stationary Gaussian processes with explicitly given covariances and cross-covariances. I will provide an integral representation for each row. The results obtained are new even for Karlin's occupancy scheme.

The talk is based on a joint work with A. Iksanov (Kyiv). This paper is available at https://arxiv.org/abs/2203.08918.

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